PULSATING SUBMERGED JET AND THERMALLY INDUCED OSCILLATIONS

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Viscous fluid flow periodically ejected from a submerged open-ended tube into a large pool and sucked back into the tube is investigated. It is shown that the resulting vortices create thermally induced oscillations in the tube.

The properties of a viscous fluid flow periodically ejected from the submerged orifice of a tube into a large pool and sucked back into the tube (Fig. 1) are of practical significance as a technique for the generation of thermally induced oscillations in nonuniformly heated ducts of engineering equipment or devices for the intensification of heat and mass transfer.

For a jet issuing from an infinitesimally narrow slot source with a harmonically varying fluid flow rate the cyclic frequency of the oscillations ω and the volume of displaced fluid referred to the slot width, $b^2 = a h_0$, are known (a is the amplitude of the oscillations of the fluid column in the slotted tube, and h_0 is the height of the slot). Here, as the linear scale we can use the quantities b, $d = \sqrt{\nu/\omega}$, or the local value of the radial coordinate r. Then, according to the Navier—Stokes equations, in a radial coordinate system r, φ (Fig. 1) with pole at the orifice of the slot the stream function $\Psi = b^2\omega\Phi$ is given by the equation

$$\begin{Bmatrix} b^4 \\ d^4 \\ r^4 \end{Bmatrix} \Delta^2\Phi + R \begin{Bmatrix} b^4 \\ d^4 \\ r^4 \end{Bmatrix} r^{-1}(\Phi_r\Delta_\varphi\Phi - \Phi_\varphi\Delta_r\Phi) - \begin{Bmatrix} Rb^4 \\ d^4 \\ I^2r^4 \end{Bmatrix} \Delta\Phi_\tau = 0, \tag{1}$$

where

$$\Delta = \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}; \quad \Delta^2 = \Delta(\Delta); \quad u = \frac{1}{r}\Psi_\varphi; \quad v = -\Psi_r;$$

$I = \sqrt{r^2\omega\nu^{-1}}$ is the local inertia number; $R = b^2\omega\nu^{-1} \sim a\omega$, Reynolds number; $\tau = \omega t$, dimensionless time; and the subscripts indicate the derivatives of functions and operators with respect to the corresponding coordinate. For a jet issuing from a slot into a channel whose walls form a flare angle $2\varphi_*$,

$$\Phi_\varphi(\varphi = \pm\varphi_*) = 0, \quad \Phi_{\varphi\varphi}(\varphi = 0) = 0; \quad \int\limits_{-\varphi_*}^{+\varphi_*} \Phi_\varphi d\varphi = 2\Phi(\varphi_*) = \cos\tau. \tag{2}$$

Accordingly, we can now indicate variants of the various asymptotic solutions of the problem. For example, the last variant for $I = 0$, $R = u_0h_0\nu^{-1} \gtreqless 0$ describes steady flows with a given flow rate u_0h_0, which are self-similar with respect to r.

For $\varphi_* = \pi/2$ in the case of small and moderate Reynolds numbers $R \ll 48\pi \approx 150$, in a zone not too far removed from the orifice of the slot, where $I^2 \ll 10$, by successive approximations $\Phi^{(1)}$, $\Phi^{(2)}$,... we obtain

$$u^{(1)} = \frac{b^2\omega}{r}\Phi_\varphi^{(1)} = \frac{b^2\omega}{r\pi}(\cos 2\varphi + 1)\cos\tau,$$
$$v^{(1)} = 0, \quad J = \rho\int\limits_{-\pi/2}^{\pi/2}[u^{(1)}]^2\, r d\varphi \infty r^{-1}, \tag{3}$$

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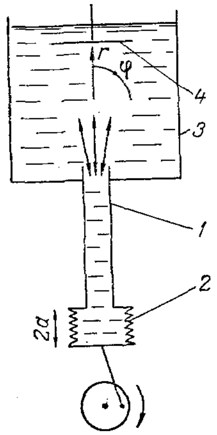

Fig. 1

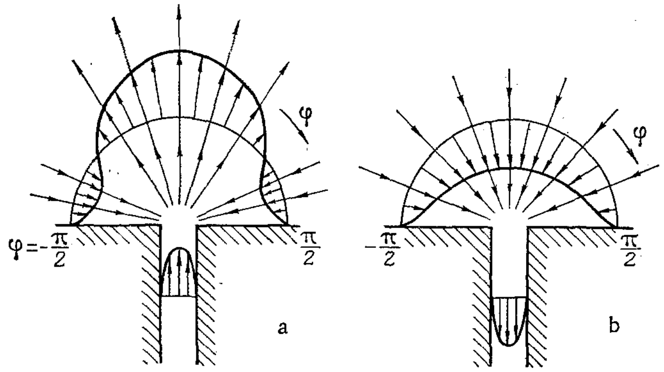

Fig. 2

Fig. 1. Test stand for the investigation of a pulsating submerged jet. 1) Glass tube; 2) metal sylphon bellows; 3) transparent vessel; 4) reflector.

Fig. 2. Velocity profiles in the tube and in free space. a) Forward flow ("exhalation"); b) reverse flow ("inhalation").

$$u_{20} = \frac{b^2\omega}{48\pi^2 r} R(6 + 7\cos 2\varphi + \cos 4\varphi - 12\varphi \sin 2\varphi) \approx u_{22},$$

$$u_{21} = \frac{0{,}2b^2\omega}{\pi r} I^2 \left[\cos 2\varphi + (\operatorname{sh}\varphi \sin\varphi - \operatorname{ch}\varphi\cos\varphi)\left(\operatorname{sh}\frac{\pi}{2}\right)^{-1}\right],$$

$$v_{21} = \frac{0{,}2b^2\omega}{\pi r} I^2 \left[-\sin 2\varphi + \left(\operatorname{sh}\frac{\pi}{2}\right)^{-1} 2\operatorname{sh}\varphi\cos\varphi\right], \quad v_{20} = 0. \tag{4}$$

In a zone far from the orifice of the slot, where $I^2 \gg 10$, for $R \ll 150$ the influence of the convection terms in (1) is lessened, while the inertial term has the effect of smoothing the velocity profile and decreasing the difference between the forward and reverse flows.

In the case of a jet issuing periodically from a thin tube the frequency ω and the volume of fluid displaced and sucked back $b^3 = 0.25a\pi D^2$, are known, so that $R = b^2\omega\nu^{-1} \propto a^{2/3}\omega$ and again $I = \sqrt{r^2\omega\nu^{-1}}$. For $\omega = 0$ even steady flows are not self-similar with respect to r, and they do not have a governing Reynolds number, in place of which is used the local number $0.25u_0\pi D^2 r^{-1}\nu^{-1}$, expressed in terms of the given fluid flow rate $0.25u_0\pi D^2$; the radial flow velocity decreases more rapidly with increasing r than in the slot case, and this fact compensates for the converging circumferential flows. The expressions for the pulsating flows are more complex here, but they essentially have the same character as the flows (3) and (4), and so we omit the corresponding expressions.

The superposition of the secondary flow components (4) onto the main component (3) creates an appreciable difference between the velocity profiles of the forward and reverse jets during "exhalation" from the orifice of a slot and during "inhalation" (Fig. 2). In the forward jet the main flow occurs in the middle zone, where the slot flow continues, whereas in the reverse jet extensive peripheral parts of the liquid are entrained into the motion. After emerging from the tube the flow ostensibly splits; some particles are sucked back into the tube after a half-period, while others escape to the periphery; the reverse flow, on the other hand, is replenished with fresh particles at the temperature of free space. This fact induces considerable intensification of the heat-transfer processes in nonuniformly heated engineering equipment or components and promotes thermally induced oscillations in them.

In the case of steady jets ($I = 0$ or $t = \text{const}$) with any flow velocities the textbook and scientific literature [1, 2] usually relies on the method of Schlichting, where only one of the Navier–Stokes equations is considered:

$$uu_x + vu_y = \nu u_{yy} \quad \text{or} \quad \Psi_y\Psi_{xy} - \Psi_x\Psi_{yy} = \nu\Psi_{yyy}$$

(in Cartesian coordinates with the x axis along the jet axis) under the extremely artificial

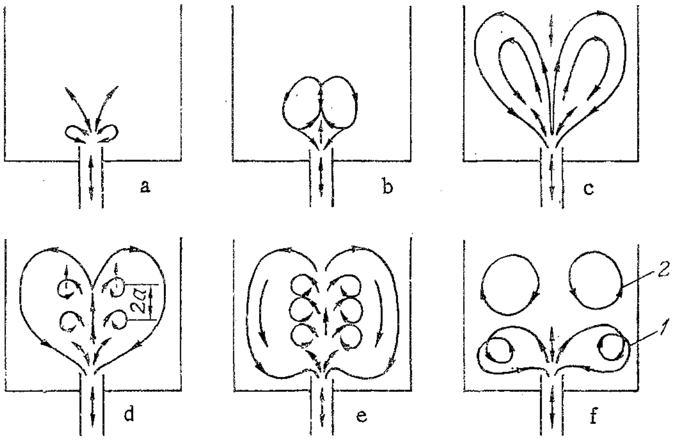

Fig. 3. Observed flow regimes in the propagation of a pulsating submerged jet.

assumption of a constant, presumably known value of the momentum $J=\rho\int_{-\infty}^{\infty}u^2dy=\rho u_0^2 h_0^2$, violating the continuity condition for real pools of finite dimensions. Accordingly, for the forward jet we obtain, with a certain degree of justification, a solution that does not contain countercurrent flows, where the velocity is concentrated in the central part of the jet ($y\approx 0$). However, if the indicated equation is solved for the reverse flow entering the duct, we obtain the altogether improbable comb velocity profile

$$u \propto [\cos^2(0.276yx^{-2/3}J^{1/3}\nu^{-2/3}\rho^{-1/3})]^{-1}x^{-1/3}$$

with minimum on the axis $y = 0$ and with a set of singularities, at which the velocity is infinite. On the other hand, it is well known that the reverse jet has a much flatter velocity profile than the forward flow. The inconsistency between the experimental data and the constant-momentum hypothesis has only recently been brought to attention [3].

For the large Reynolds numbers R and arbitrary not too small φ_* normally realized in practice, as well as for $\varphi_* > \pi/2$ and arbitrary R, the solutions of Eq. (1) invariably contain large transverse velocity components and can be obtained only by computer techniques. Such solutions contain the inverse of the velocity and are not too stable. It is well known that real steady submerged forward jets contain a diminishing laminar column on the flow axis, surrounded by turbulent vortices in the expanding conical zone. Reverse steady flows are highly stable, and the laminar regime is preserved even for relatively large Reynolds numbers. It is also known that relatively stable vortex rings are formed in the impulsive ejection of fluid from the orifice of a tube [4]. The pulsating submerged jet contains elements of these familiar flows in some measure. The presence of the squared terms in the middle part of Eq. (1) stipulates the existence of steady circulating flows for small [solution (4)] as well as for large Reynolds numbers. A certain role is also taken in real devices by the difficult-to-analyze details of the flow around the tube orifice (slot) in a zone commensurate with its diameter D (height h_0), where pipe flow ostensibly continues, a fact that is overlooked by the boundary conditions (2). Besides the numbers I and R, the behavior of pulsating jets in the vicinity of the orifice is also affected by the inertial number $I_D=0.5D\sqrt{\omega\nu^{-1}}$ and Reynolds number $R_D=a\omega D\nu^{-1}$, evaluated along the tube diameter.

Because of the complexity of the calculations, we have investigated pulsating submerged jets experimentally. We constructed a test stand (Fig. 1) consisting of a glass tube ($D = 0.57$ cm), one end of which opens into a wide vessel 3, while the other is connected to a periodically compressed sylphon bellows. In the experiments we used water or glycerin and solutions of the latter with viscosities $\nu = (1.1\text{-}710)\cdot 10^{-6}$ $m^2\cdot sec^{-1}$; the flows were visualized by the addition of aluminum powder. The amplitude and cyclic frequency of the oscillations of the liquid column in the tube were $a = 0.3\text{-}3$ cm and $\omega = 4\text{-}26$ sec^{-1}, with $I_D = 0.24\text{-}13$ and $R_D = 0.2\text{-}3000$. The Reynolds number for the free volume was

$$R=\left(a\frac{\pi D^2}{4}\right)^{2/3}\omega\nu^{-1}=\pi^{2/3}4^{-1/3}I_D^{2/3}R_D^{2/3}=0.2\text{—}1500.$$

The visual observations were accompanied by photographic recording. The flow in the tube remained laminar at all times. This fact was evident in the motion of the tracer particles, and the observed flow regimes were far from the stability limits $A = 2a\sqrt{\omega\nu^{-1}} < A_* = 700$, determined by Sergeev [5]. The experiments were carried out both with tubes installed flush with the bottom of the vessel 3 ($\varphi_* = \pi/2$) and with tubes projecting up from the bottom ($\varphi_* = \pi$). This modification was observed to create a difference in the flow only for large

Reynolds numbers $R > 200$, in which case liquid beneath the orifice of the tube was entrained in the motion.

The following regimes were observed:

a) For small Reynolds numbers $R < 5$ with $\varphi_* = \pi/2$ and $\varphi_* = \pi$ the pulsating submerged jet in both the forward ("exhalation") and the reverse ("inhalation") directions has practically the same configuration of a slightly divergent hyperboloid or plume (Fig. 3a), and the velocity profiles of the forward and reverse jets are close to the profiles (3) and (4), with only a slight violation of the piston analogy.

b) With an increase in the Reynolds number the piston analogy is clearly violated, and for $R \sim 20$–40 a stable flow emerges for which at the "exhalation" times liquid breaks away from the tube in the form of a plume and generates a single well-defined almost-spherical toroidal vortex or circulation flow near the tube orifice (Fig. 3b) with a volume approximately twice the volume amplitude of the liquid pulsations $0.25\pi D^2 a$. At just a short distance from the boundaries of the vortex (of the order of 0.1 times its radius) the velocity of the liquid decreases abruptly. In the reverse direction ("inhalation") a portion $(1 - k)$ of the ejected liquid returns to the tube from the piston zone (plume), while the other portion (k) escapes from the periphery of the vortex, where the circulation velocity is constantly directed toward the tube orifice. The mass-transfer coefficient k is difficult to measure directly. We had to develop a device of the local flowmeter type, which integrates the local motion of a large number of particles. We estimated k in our experiments in the course of repeated careful visual observations of the motion of tracer particles with the use of a stopwatch, within approximately ±15% error limits. In this regime $k \sim 0.1$–0.15.

c) With a further increase in R ($R \sim 60$–120) the volume occupied by the vortex increases, the boundaries of the vortex become diffuse, and it is deformed along the jet axis, setting the stage for breakup of the vortex and its detachment from the tube orifice (Fig. 3c). The steady circulation trajectory and transit time of particles over this trajectory also grow simultaneously, so that the indicated time attains a value of the order $10t_* = 20\pi\omega^{-1}$. In this regime $k \approx 0.25$–0.3.

d) With a further increase in R, $R \sim 170$–420, the observed motion contains several intermingled vortices. The vortices nucleate in the orifice zone of the tube at the times of maximum slowing down of the liquid ejected from the tube, grow at the start of the path, and are then carried by the flow in the direction of propagation of the jet (Fig. 3d). In addition, these vortices are surrounded by a singular steady circulation flow. This ordered system is an important distinction of the pulsating jet relative to a steady jet, where the vortices are almost totally disordered. The singular train of regular vortices emanating from the orifice with a spacing $\sim 2a$ is very distinctly observed and is highly typical of this regime. Here the coefficient k is roughly equal to 0.5.

e) With a greater increase in the pulsation intensity ($R \sim 420$–620) the vortices grow in size to the point of almost merging with one another, forming an almost ordered cellular structure (Fig. 3e). The trajectory of the surrounding circulation is still broader. Also, the interaction of the countercurrent flows in the vortices is not too strong and has not yet disrupted the vortex system. The mass-transfer coefficient k in this case attains a maximum value of 0.6–0.75.

f) At a greater pulsation intensity ($R > 620$) the vortices interact with one another and penetrate the steady circulation flow. This flow breaks up into several self-contained circulations (Fig. 3f), the number of which increases with a further increase in the pulsation intensity. During suction of the liquid back into the tube, particles arrive from the nearest, relatively contracted ring (1, Fig. 3f), whereas in the more distant rings (2, Fig. 3f) circulation takes place without any clearcut mass transfer with the particles of liquid leaving and returning to the tube. The positions of the vortices become variable, and such motions, while preserving elements of order, also acquire a random character.

For practical applications we carried out another series of experiments with a metal reflector 4 installed in the vessel 3 (Fig. 1). It was found that the circulation motion becomes stronger as the reflector is brought nearer the open end of the tube. If the distance from the reflector to the tube orifice is $\sim 4a$, it promotes transition from regime c) at low pulsation intensities to regime d) or e), greatly intensifying the mass transfer associated with particle ejection and suction. When the reflector is very close to the orifice, at a distance less than $2a$, it mainly elicits a change in the direction of the jet. Here the vortices are broken up as in regime f).

In cryogenic-engineering equipment one often encounters nonuniformly heated tubes with a relatively warm closed end and an open end entering a tank containing helium vapor and sometimes liquid helium. Self-excited oscillations of the helium vapor are easily generated in such tubes and are sustained by a steady flux of thermal energy; they are called thermally induced oscillations [6]. In this case the tube operates like a steam engine, except that there are no valves, the role of which is taken by the vortices and mass transfer near the open end of the tube. Here, under the conditions of maximum intensity of the thermally induced oscillations, motions corresponding to regime e) are observed [7]. A measure of the power of such a heat engine is the enthalpy transfer $k\Delta T$ per cycle, where ΔT is the temperature difference between the gas ejected from the tube and the surrounding gas near the tube orifice. The incipient thermally induced self-excited oscillatory motions of the gas are self-intensified in the initial stage, producing a sharp burst of oscillations. Thereafter (with development of the oscillations) the quantity $k\Delta T$ not only does not increase, it even decreases as a result of transition to regime f) with a contracted steady-circulation trajectory and reductions of ΔT and k. The excitation of oscillations is suppressed by equalization of the temperature of the gas inside the tube and in the surrounding near space, whereas friction continues to grow. The thermally induced self-excited oscillations are not longer amplified, and they settle into self-excited oscillations with a limit cycle. This explanation accounts for the previously known experimental fact [6] of thermally induced oscillations with a steady-state amplitude.

According to [6], the thermally induced oscillations of helium in nonuniformly heated tubes whose orifices are close to the surface of liquid helium in a tank become particularly strong when the distance from the surface of the liquid to the orifice of the tube is about 2-8 cm. The experiments with the reflector, which functions as a liquid surface in connection with thermally induced oscillations, explain this effect, as well as the considerable attenuation of the oscillations as the tube is brought closer to the liquid-helium tank. Experiments on the test apparatus described in [6] have confirmed the fact that, because of the intensification of heat transfer in connection with thermally induced oscillations, the volatility in liquid-helium equipment can exceed the normal value by a factor of 200 in individual tests due to heat inputs via the gas column and heat bridges as well as through the thermal insulation. By monitoring the intensity of the thermally induced oscillations according to the position of a tube relative to the liquid surface or the displacement of special baffles it is possible to admit whatever heat is technically necessary in order to evaporate cryogenic liquids. For this reason we have carried out studies on the useful regulated injection of heat into cryogenic devices for generation of the charging gas in them. Other applications of the rotational mass transfer in a pulsating submerged jet are also possible.

NOTATION

A, amplitude number; a, amplitude; D, diameter; d, a linear scale; h_0, height; I, inertia number; J, momentum; k, mass-transfer coefficient; R, Reynolds number; r, radial coordinate; T, temperature; t, time; u, radial velocity; v, circumferential velocity; ν, kinematic viscosity; ρ, density; Φ, dimensionless stream function; φ, circumferential coordinate; Ψ, stream function; ω, cyclic frequency.

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